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New ranking function for fuzzy linear programming problem and system of linear equations

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Abstract

Linear Programming problems and System of Linear equations have many applications in various science and engineering problems like network analysis, operations research etc. In general Linear Programming Problem (LPP) and the system of linear equations contain crisp parameters that is real numbers or complex numbers as their coefficients and constants, but in real life applications, LPP and system of equations may contain the constrains or the parameters as uncertain. These uncertain values are not the exact real numbers but vary within some range of values, the values may vary within an interval or can be considered as fuzzy number.

In this paper, we have developed a new Ranking function (which converts the fuzzy number into crisp) to solve a fully fuzzy LPP and System of equations. Unlike the previous ranking functions, the proposed ranking function uses fuzzy number itself improving the accuracy of the solution. The ranking function is derived by replacing the non-parallel sides of the trapezoidal fuzzy number with non-linear functions. Various numerical examples are included and compared with the pre-existing methods.

Subject Classification: *03E72, 90C70, 65H10, 90C05.*

Keywords: *Fuzzy Set Theory, Fuzzy Programming, System of equations, Linear programming*

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1. Introduction

The lack of certainty in the information gives rise to the concept of Interval or Fuzzy set theory. For example, it is unknown whether it rains tomorrow; this kind of situation is called Uncertainty. If probability is applied to check the possible outcomes, the uncertainty can be known. Suppose it says 85% of sunshine, then there is 15% chance of raining. But there is always some error in calculating the possibility. If the error percentage is 15% then the probability of raining is in between [15-15% ,15+15%], this is in the Interval form, i.e., the possibility of raining tomorrow varies from a certain value to a certain value and there exists infinite quantities in that interval. Therefore, it is not advisable to consider one single value to predict the outcome. Hence the concept of Fuzzy is being introduced.

System of linear equations and Linear programming problems have various applications like network analysis, curve fitting, operation research, banking and finance etc. Equations of this type are necessary to solve for getting the involved parameters. It is simple and straight forward when the variables involving the system of equations are crisp. But in actual case the system variables may not be obtained as crisp. Those are found by conducting some experiment in general. So, these variables will either be an interval or a fuzzy number, for example, the measurement of the length of a wire does not give the crisp value in particular. As such there will be vagueness in the result of the experiment. To overcome the vagueness we may use the interval and fuzzy numbers in the place of crisp number [1].

Fuzzy linear systems had recently been studied by many authors but most of the studies include only constants as fuzzy or the variables as fuzzy. In this paper, we transform all the fuzzy numbers into real numbers by using the ranking function and solve the linear equations. There are few authors who have given various ranking functions in fuzzy numbers like weighted distance ranking function[1],Yager's ranking function[2], Maleki's ranking function[3], Area ranking function[4] etc. We concentrate on Yager's [2] ranking function formula and derive the new ranking function which is proved to be more efficient by comparing the results of different examples. Further Tang et al. [17] proposed a weighted ranking function for ranking triangular fuzzy numbers.

Since the introduction of the concept of Maximizing set to compare and arrange the fuzzy numbers in 1976 by Jain [5, 6] many authors have introduced various ranking methods.

There are methods which use centroid of the fuzzy number to compare the fuzzy numbers [6], and also weighted distance methods [1], and using the λ-cuts of the fuzzy number [7]. Maleki et al. [3] derived a ranking function formula using linear fuzzy number. Yager in 1981 [2] has introduced a ranking function formula to solve a fuzzy linear programming problem using non-linear fuzzy number. The formula given by Yager[2] is used in our current paper but with different functions. The functions used by Maleki[3] and Yager[2] do not geometrically support the fuzzy number existence. Liang et al. [18] investigated the application of fuzzy linear programming to project crashing decisions where they considered the variables as fuzzy.

A new ranking function is proposed here using the ranking formula proposed by Yager[2] with non-linear functions on the left and right ends of the L-R flat fuzzy number. The non-linear functions introduced are parabola with their vertices as part of the fuzzy number, which makes the derived ranking function more efficient in comparing two fuzzy numbers.

When two real numbers are compared, we check which one is bigger or smaller than the other. In the same way, two fuzzy numbers are compared by the size (corresponding crisp value which is obtained by applying ranking function formula) of the fuzzy numbers. This can be explained geometrically well and the correctness of the comparison operator is checked by geometrical comparison of the fuzzy numbers.

In this paper, we have first discussed about fuzzy number and its arithmetic and λ -cut of a fuzzy number. The concept of these has been used for the numerical solution of system of linear equations and linear programming problems. As such new method is developed here to handle these fuzzy problems in $[0, \infty]$. In special cases the solutions are also compared with the known results that are found in literature.

2. Preliminaries

In this section we include some basic definitions.

Definition 2.1: (Classical Set)

A Set is a well-defined collection of objects.

Classical, or a crisp set, is one which assigns grades of membership of *either 0* or *1* to objects.

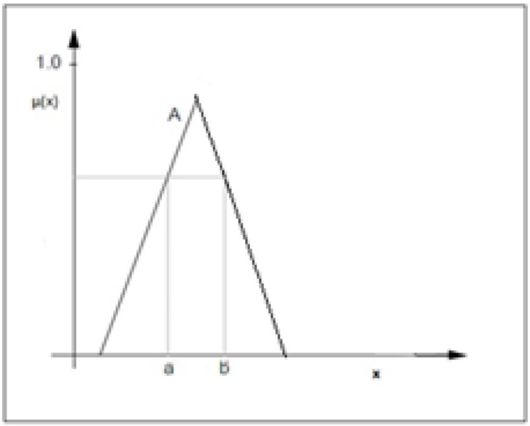

Figure 1

Illustration of an λ-cut

Definition 2.2: (Fuzzy Set)

A fuzzy set can be defined as the set of ordered pairs such that $A = \{(x, \mu(x))/\ x \in \mathbf{X}, \mu(x) \in [0, 1]\}$ where $\mu(x)$ is called the membership function.

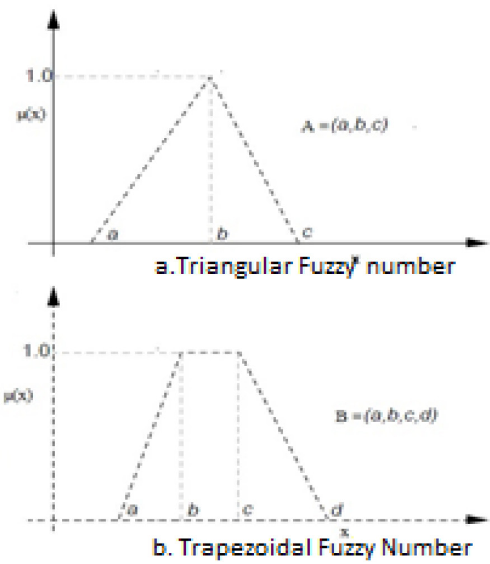

Figure 2

Types of Fuzzy numbers

Definition 2.3: (λ-cut of a fuzzy set)

λ-cut of a fuzzy set is the crisp set denoted by A_λ (a crisp interval), for a particular value of membership value λ, $A_\lambda = [a, b]$ as shown in the figure 1, and λ is in [0, 1]

Definition 2.4: (Types of Fuzzy Numbers)

Most commonly used fuzzy numbers are Triangular and Trapezoidal fuzzy numbers, there are other types of fuzzy numbers like flat fuzzy numbers, which will be used in this report further. The triangular fuzzy number is defined using 3 parameters and Trapezoidal fuzzy number is defined by 4 parameters, which are explained in the following figure,

Definition 2.5: (Fuzzy Arithmetic)

Generally, a fuzzy number is converted into its λ-cut and the arithmetic operations are carried on as of for an interval,

2.5.1 Interval Arithmetic

For any two intervals $[a, b]$ and $[d, e]$, the arithmetic operations are performed in the following way-

a. **Addition:** $[a, b] + [d, e] = [a + d, b + e]$;
b. **Subtraction:** $[a, b] - [d, e] = [a - e, b - d]$;
c. **Multiplication:** $[a, b] \cdot [d, e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)]$;
d. **Division:** $[a, b] / [d, e] = [\min(a/d, a/e, b/d, b/e), \max(a/d, a/e, b/d, b/e)]$, provided 0 is not in $[d, e]$.

Definition 2.6: ($L - R$ Flat Fuzzy Number)

A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be an L-R Flat fuzzy number if,

$$\mu(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right), & x \le m, \quad \alpha > 0 \\ R\left(\dfrac{x-n}{\beta}\right), & x \ge n, \beta > 0 \\ 1, & \text{else} \end{cases}$$

Where L and R are called reference functions, which are continuous non increasing functions that define the left and right shapes of membership function and $L(0) = R(0) = 1$.

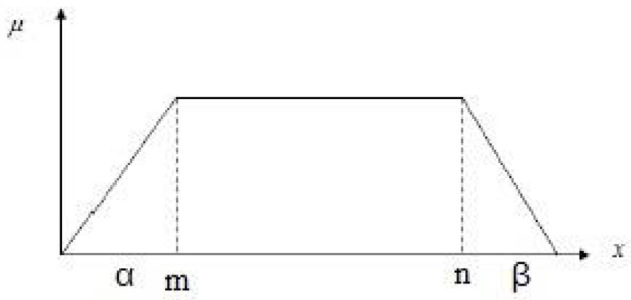

Figure 3

L-R Flat Fuzzy Number

2.7 *Arithmetic Operations of L-R Flat fuzzy number:*

Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)$ be two non-negative L-R flat fuzzy numbers, then the arithmetic operations are defined as follows:

(1) Addition/Subtraction:

$$\begin{aligned}\tilde{A}_1 \pm \tilde{A}_2 &= (m_1, n_1, \alpha_1, \beta_1) \pm (m_2, n_2, \alpha_2, \beta_2)\\ &= (m_1 \pm m_2, n_1 \pm n_2, |\alpha_1 \pm \alpha_2|, |\beta_1 \pm \beta_2|)\end{aligned}$$

(2) Multiplication: (for $m_1 - \alpha_1 > 0$ and $m_2 - \alpha_2 > 0$)

$$\begin{aligned}\tilde{A}_1 \times \tilde{A}_2 &= (m_1, n_1, \alpha_1, \beta_1) \times (m_2, n_2, \alpha_2, \beta_2)\\ &= (m_1 m_2, n_1 n_2, m_1\alpha_2 + m_2\alpha_1, n_1\beta_2 + n_2\beta_1)\end{aligned}$$

(3) Scalar Multiplication:

$$\begin{aligned}k\tilde{A}_1 &= (km_1, kn_1, k\alpha_1, k\beta_1),\ k \geq 0\\ k\tilde{A}_1 &= (km_1, kn_1, -k\beta_1, -k\alpha_1), k \leq 0.\end{aligned}$$

Definition 2.8: (λ - cut of *L-R* Flat fuzzy Number)

Let $\tilde{A} = (m, n, \alpha, \beta)$ be an *L-R* flat fuzzy number and α be a real number in the interval [0, 1] then the crisp set

$$\begin{aligned}A_\lambda &= \{x\, \varepsilon\, X : \mu(x) \geq \lambda\}\}\\ &= [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]\end{aligned}$$

is said to be λ - cut of $\tilde{A}$.

A *L-R* flat fuzzy number is said to be non-negative if $m - \alpha \geq 0$.

Definition 2.9: (Ranking Function)

The concept of ranking function was introduced to compare two fuzzy numbers, which is practically not possible with four coefficients. Hence, the fuzzy number is converted into real number and then the fuzzy numbers are compared.

A ranking function $R_a : F(R) \rightarrow$ R, maps each fuzzy number into a real line. Now, suppose that $\tilde{A}$ and $\hat{A}$ be two fuzzy numbers. We define R_a as follows:

1. $\tilde{A} \geq \hat{A}$ if and only if $R_a(\tilde{A}) \geq R_a(\hat{A})$
2. $\tilde{A} > \hat{A}$ if and only if $R_a(\tilde{A}) > R_a(\hat{A})$
3. $\tilde{A} = \hat{A}$ if and only if $R_a(\tilde{A}) = R_a(\hat{A})$
4. $\tilde{A} \leq \hat{A}$ if and only if $\hat{A} \geq \tilde{A}$

2.10. Yager's Ranking Function

Yager proposed for ordering fuzzy sets in which a ranking index $R_a(\tilde{A})$ is calculated for the fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ for its λ - cut

$A_\lambda = [m\text{-}\alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$ according to the formula

$$R_a(\tilde{A}) = \frac{1}{2}\int_0^1 ((m - \alpha L^{-1}(\lambda)) + (n + \beta R^{-1}(\lambda)))\,d\lambda \qquad (1)$$

where $L(x) = \max(0, (1 - x^4))$ and $R(x) = \max(0, (1 - x^2))$. Then by using equation (1) we have

$$R_a(\tilde{A}) = \frac{1}{2}\left(m + n - \frac{4}{5}\alpha + \frac{2}{3}\beta\right)$$

2.11. *Maleki's Ranking Function*

Here the formula is given as

$$R_a(\tilde{A}) = m + n + \frac{1}{2}(\beta - \alpha)$$

where $L(x) = \max(0, (1 - x))$ and $R(x) = \max(0, (1 - x))$

3. Proposed Ranking Function:

We introduce here a new ranking function with $L(x)$ and $R(x)$ as monotonically increasing and decreasing non-linear functions, say

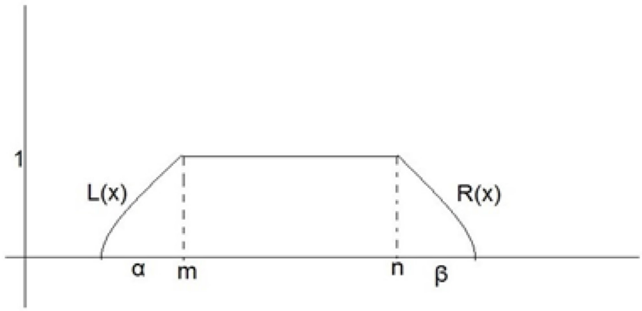

Figure 4

***L-R* Flat Fuzzy Number with Non-Linear Functions**

parabolas. Then using Yager's Ranking formula, equation (1), we have derived a new ranking function as below.

For a fuzzy number $\tilde{A}$ = (m, n, α, β), the ranking function can be derived using the formula proposed by Yager as in equation(1)

$$R_a(\tilde{A}) = \frac{1}{2}\int_0^1((m-\alpha L^{-1}(\lambda))+(n+\beta R^{-1}(\lambda)))d\lambda$$

Where λ in [0, 1]

Let us insert parabolas as $L(x)$ and $R(x)$ and consider the fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$, let $m = b$, $n = c$, $m - \alpha = a$, $n + \beta = d$

Also Let us suppose that $L(x)$ be a parabola with vertex (a, 0) and passing through (b, 1) and $R(x)$ be another parabola with vertex (d, 0) and passing through (c, 1) which are shown in Figure (4)

Accordingly we get,

$$L(x) \text{ as } y^2 = \frac{x-a}{b-a},$$

$$L(x) \text{ as } y^2 = \frac{x-d}{c-d}$$

Let $L(x) = \max\left(0, \sqrt{\frac{x-a}{b-a}}\right)$ and $R(x) = \max\left(0, \sqrt{\frac{x-d}{c-d}}\right)$

To derive the new ranking function, first we find the inverse of the above functions

We obtain,

$$L^{-1}(x) = a + x^2 * (b-a) \quad (2)$$

And $$R^{-1}(x) = d + x^2 * (c-d) \quad (3)$$

Now, substituting equations (2) & (3) in equation (1) , we get,

$$R_a(\tilde{A}) = \frac{1}{2}\int_0^1 \Big(\big(m-\alpha L^{-1}(\lambda)\big)+\big(n+\beta R^{-1}(\lambda)\big)\Big)d$$

$$= \frac{1}{2}(b-a(b-a)-\frac{(b-a)^2}{3}+c+d(d-c)-\frac{(d-c)^2}{3}$$

Substituting $b = m$, $b - a = \alpha$, $c = n$, $d - c = \beta$ one obtains,

$$R_a(\tilde{A}) = \frac{1}{2}\left(m(1-\alpha)+n(1+\beta)+\frac{2}{3}\alpha^2+\frac{2}{3}\beta^2\right)$$

Above is the proposed new Ranking function.

3.1 *Comparison with previous Ranking Functions:*

The same methodology was applied for the following two pre-existing ranking functions and the results were found to be in concurrence. Thus, the methodology was proved to be successful.

Theorem 3.1.1 [3]: *(Maleki's Ranking function)*

For an LR- flat fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$, with λ - cut $A_\lambda = [m-\alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$, linear functions are introduced on right and left ends of the trapezoidal fuzzy number i.e., $L(x) = R(x) = \max(0, 1 - x)$.

Proof: Using the Yager's Ranking formula[1], we have from equation (1)

$$R_a(\tilde{A}) = \frac{1}{2}\int_0^1 \Big(\big(m-\alpha L^{-1}(\lambda)\big)+\big(n+\beta R^{-1}(\lambda)\big)\Big)d\lambda$$

Now to derive Maleki's ranking formula, first the inverse functions are derived,

We get, $L^{-1}(\lambda) = 1 - \lambda$

& $R^{-1}(\lambda) = 1 - \lambda$

Substituting these in the Yager's ranking formula, equation(1), we get Maleki's ranking formula as,

$$R_a(\tilde{A}) = \frac{1}{2}(m+n+\frac{1}{2}(\beta-\alpha)$$

Theorem 3.1.2 [2]: *(Yager's Ranking Formula)*

For an LR-flat fuzzy number, $\tilde{A} = (m, n, \alpha, \beta)$, with λ - cut $A_\lambda = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$, non-linear functions are introduced on right and left ends of the trapezoidal fuzzy number i.e., $L(x) = \max(0, (1 - x^4))$, $R(x) = \max(0, (1 - x^2))$.

Proof: Using the Yager's Ranking formula, from equation (1)

$$R_a(\tilde{A}) = \frac{1}{2}\int_0^1 \left(\left(m - \alpha L^{-1}(\lambda)\right) + \left(n + \beta R^{-1}(\lambda)\right)\right) d$$

Now to derive Yager's ranking formula, first the inverse functions are derived.

We get,

$$L^{-1}(x) = (1-x)^{\frac{1}{4}} \text{ and } R^{-1}(x) = (1-x)^{\frac{1}{2}}$$

Substituting these in equation (1) we get the Yager's ranking function [2] as,

$$R_a(\tilde{A}) = \frac{1}{2}\left(m + n - \frac{4}{5}\alpha + \frac{2}{3}\beta\right)$$

3.2 *Examples*

Example 1: Let $\tilde{A}$ = (01, 2, 2, 1) and $\hat{A}$= (–4, –2, 1, 1)

Example 2: Let $\tilde{A}$ = (4, 8, 2, 2) and $\hat{A}$ = (1, 3, 1, 1)

Example 3: Let $\tilde{A}$ = (1, 5, 2, 2) and $\hat{A}$ = (1, 7, 3, 3)

Example 4: Let $\tilde{A}$ = (4, 8, 3, 1) and $\hat{A}$ = (1, 5, 3, 1)

Example 5: Let $\tilde{A}$ = (5, 8, 2, 5) and $\hat{A}$ = (6, 10, 2, 6)

Example 6: Let $\tilde{A}$ = (13, 15, 2, 2) and $\hat{A}$ = (12, 14, 3, 3)

Example 7: Let $\tilde{A}$ = (3, 4, 2, 3) and $\hat{A}$ = (3, 5, 1, 4)

Example 8: Let $\tilde{A}$ = (3, 6, 1, 2) and $\hat{A}$ = (2, 4, 1, 2)

Example 9: Let $\tilde{A}$ = (5, 6, 2, 4) and $\hat{A}$ = (7, 10, 3, 4)

Example 10: Let $\tilde{A}$ = (10, 13, 2, 4) and $\hat{A}$ = (18, 24, 3, 6)

Table 1

Comparison with previous ranking functions

Question	Yager's Ranking Function	Maleki's Ranking Function	New Ranking Function
Example 1	$\tilde{A} > \hat{A}$	$\tilde{A} > \hat{A}$	$\tilde{A} > \hat{A}$
Example 2	$\tilde{A} > \hat{A}$	$\tilde{A} > \hat{A}$	$\tilde{A} > \hat{A}$
Example 3	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$
Example 4	$\tilde{A} > \hat{A}$	$\tilde{A} > \hat{A}$	$\tilde{A} = \hat{A}$
Example 5	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$
Example 6	$\tilde{A} > \hat{A}$	$\tilde{A} > \hat{A}$	$\tilde{A} < \hat{A}$
Example 7	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$
Example 8	$\tilde{A} > \hat{A}$	$\tilde{A} > \hat{A}$	$\tilde{A} > \hat{A}$
Example 9	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$
Example 10	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$	$\tilde{A} < \hat{A}$

One may see from Table 1 that Examples 4 and 6 give different conclusion about comparison of two fuzzy numbers. The size of the fuzzy number or the area of the trapezoid is the mode of comparison given in [4]. Let us give geometrical representation of fuzzy numbers mentioned in Table 1 of Examples 4 and 6

Example 4: $\tilde{A}$ = (4, 8, 3, 1) and $\hat{A}$= (1, 5, 3, 1)

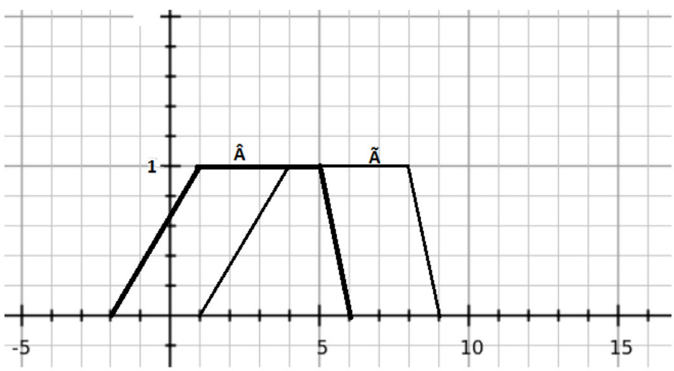

Figure 5

Comparison 1

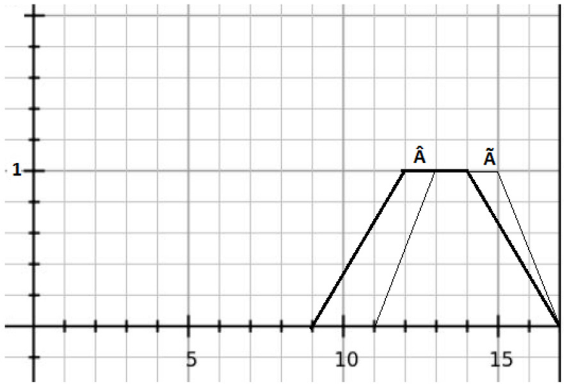

Figure 6

Comparison 2

If we consider figure-5, the *L-R* flat fuzzy numbers $\tilde{A}$= (4, 8, 3, 1) and $\hat{A}$ = (1, 5, 3, 1) are shown in thin and bold lines respectively. Considering area wise, both the fuzzy numbers are congruent to each other.

Example 6: $\tilde{A}$ = (13, 15, 2, 2) and $\hat{A}$ = (12, 14, 3, 3)

From figure-6 it is clear that $\tilde{A}$ = (13, 15, 2, 2) is geometrically smaller than $\hat{A}$ = (12, 14, 3, 3).

Hence, the geometrical result and the result we got from using the formula we derived are the same. This explains the correctness of the formula derived.

All the above examples clearly explain that the derived ranking function is more accurate and correct to compare two fuzzy numbers.

4. Fuzzy LPP and System of Linear Equations:

4.1 *Examples of Fuzzy LPP*

Example 11: (Objective Function Fuzzy)

Max(z) = (3, 4, 3, 1) x_1 + (2, 2, 1, 2) x_2

Subject to,

$$3\,x_1 + 2\,x_2 \leq 5$$

$$5x_1 + 4\,x_2 \leq 8$$

Example 12: (Constraints Fuzzy)

Max(z) = $3x_1 + 2x_2$

Subject to,

Table 2
Comparison of Fuzzy LPP solutions

#	Our Ranking Function	Maleki's Ranking Function	Yager's Ranking function
Example 1	9.33	9	4.533
Example 2	9.334	7.7767	6.180823
Example 3	16.59154	15.7164	6.46400

$$(3, 6, 1, 2)\ x_1 + (2, 4, 1, 2)\ x_2 \leq (10, 13, 2, 4)$$

$$(5, 6, 2, 4)\ x_1 + (7, 10, 3, 4)\ x_2 \leq (18, 24, 3, 6)$$

Example 13: (Fully Fuzzy LPP)

Max(z) = (3,4,3,1) x_1 + (2,2,1,2) x_2

Subject to,

$$(3, 6, 1, 2)\ x_1 + (2, 4, 1, 2)\ x_2 \leq (10, 13, 2, 4)$$

$$(5, 6, 2, 4)\ x_1 + (7, 10, 3, 4)\ x_2 \leq (18, 24, 3, 6)$$

Solution of the above three examples in comparison of other Ranking functions are given in the following table 2.

From table 2, we can clearly say that the new Ranking function derived gives the maximum value of Objective function. Hence the proposed formula is better as Maximum estimator of the Objective function.

4.2 *Examples on System of Linear Equations:*

Example 14: (Fully Fuzzy)

$$(3, 6, 1, 2)\ x_1 + (2, 4, 1, 2)\ x_2 = (10, 13, 2, 4)$$

$$(5, 6, 2, 4)\ x_1 + (7, 10, 3, 4)\ x_2 = (18, 24, 3, 6)$$

Applying the ranking Function on left hand side:

We get the equations as

$$64x_1 + 46x_2 = (10, 13, 2, 4)$$

$$115x_1 + 158x_2 = (18, 24, 3, 6)$$

Solving this using Gauss elimination method, we get

$$x_1 = \left(\frac{5739}{36800}, 0.1970, 0.0141, 0.0282\right)$$

$$x_2 = \left(\frac{2}{4822}, \frac{41}{4822}, \frac{115}{4822}, \frac{230}{4822}\right)$$

To check the correctness of the solution, we substitute in the equations and check whether it is tolerance, algebraic or control solution:

Substituting x_1 and x_2 in $64x_1 + 46x_2$ we get

(9.999, 12.999, 1.999, 3.9989)

This is approximately equal to (10, 13, 2, 4)

Substituting x_1 and x_2 in $115x_1 + 158x_2$ we get,

(17.99990797, 23.99842596, 5.389645998, 10.779292)

When we plot this we get the control solution, but the interval in which the membership function is 1 remains the same i.e., [18, 24]

Thus we may conclude that the proposed solution is more accurate.

Hence, we got the algebraic solution using our ranking function.

Example 15: (RHS vectors as Fuzzy)

$$2x_1 + x_2 + 3x_3 = (7, 13, 6, 6)$$

$$4x_1 + x_2 - x_3 = (6, 12, 6, 6)$$

$$-x_1 + x_2 + 3x_3 = (0, 5, 5, 5)$$

Solving this by Gauss Elimination method, we get-

$$x_1 = \left(\frac{7}{3}, \frac{13}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$x_2 = \left(\frac{-28}{12}, \frac{-35}{12}, \frac{29}{6}, \frac{29}{6}\right)$$

$$x_3 = \left(\frac{17}{12}, \frac{29}{12}, \frac{1}{6}, \frac{1}{6}\right)$$

Substituting these values in

1. $2x_1 + x_2 + 3x_3$ we get (7, 13, 6, 6)
2. $4x_1 + x_2 - x_3$ we get (6, 12, 6, 6)
3. $-x_1 + x_2 + 3x_3$ we get (0, 0, 5, 5)

If we check the types of solution, we get x_3 as the **tolerance** solution and x_2, x_1 as **algebraic** solutions.

5. Conclusion

In this paper, we have proposed a new method to solve fuzzy linear programming problems and fuzzy linear system of equations. The concept of a new ranking function is introduced. The new ranking function gives the **maximized objective function** value in a fully fuzzy LPP and in case of fuzzy system of equations it gives either **algebraic solution or tolerance solution**. Linear system of equations has different types of solutions but the method proposed shows that the solution obtained satisfies the given conditions.

Moreover, the proposed method is simpler and completely efficient to handle. This can solve LPP and system of equations with partial fuzziness or fully fuzziness too.

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